

On a class of small nonlocal set of n -party orthogonal product states

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It is shown that there exists a class of locally indistinguishable, completable, small set of n -party ($n \geq 3$) orthogonal product states, with $2n(d-1)$ members in $(d)^{\otimes n}$ dimensions ($d \geq 2$), imposing the condition that no party is able to begin with a nontrivial and orthogonality preserving measurement in $d \times d$ dimensions. In fact, it is possible to construct a multipartite nonlocal orthogonal product basis in $(d)^{\otimes n}$ dimensions which contains fewer than $2n(d-1)$ such states (along with other product states) and that does not satisfy the above mentioned condition. As a result of which some information, useful for state discrimination, can be extracted without disturbing the orthogonality of the post measurement states. So, imposing such a condition, is related to the stronger form of nonlocality. Finally, it is shown that a 2×2 maximally entangled state as resource, along with a local protocol, is sufficient for perfect discrimination of such a nonlocal set of n -party orthogonal product states in $(d)^{\otimes n}$ dimensions.

I. INTRODUCTION

In quantum information theory, the problem of discriminating quantum states has got notable attention in last couple of decades. If the quantum states, to be discriminated, are orthogonal, then they can be discriminated by performing a suitable global measurement. But the task of discriminating quantum states of a given set, becomes difficult when the parties are restricted to perform only *local operations and classical communication* (LOCC). The LOCC state discrimination problem [1–8] can be defined as follows: there are two or more spatially separated observers, holding a quantum system, which is prepared in one of the states of a known set and their objective is to identify the state correctly, provided that the observers are allowed to perform any sequence of operations on their own subsystem only. The observers can also communicate with each other classically.

In many of the papers, the authors have showed that there is a clear connection between local indistinguishability and entanglement [2, 5, 9]. However, it was first shown by Bennett *et al.* that there exists an orthonormal product basis in $3 \otimes 3$, which can not be distinguished perfectly by LOCC only, in their famous paper “Quantum nonlocality without entanglement” [10]. In Ref. [14], Walgate *et al.* showed a simple proof to establish the local indistinguishability of the nine product states of Bennett *et al.* Feng *et al.* presented locally indistinguishable orthogonal product states in $2 \otimes 2 \otimes 2$ [21]. Childs *et al.* showed a way to bound nonlocality while discriminating a set of orthogonal product states, containing domino-type tilings [23]. In recent days, the construction of locally indistinguishable bipartite orthogonal product bases has got wide attention. Not only locally indistinguishable complete orthogonal product bases are introduced but also some small sets are introduced which exhibit ‘non-locality without entanglement’ [15–17, 21, 22, 24–28].

Many of them considered the construction of multipartite orthogonal product bases that are locally indistinguishable. For example, in Ref. [16] the authors constructed a class of locally indistinguishable multipartite orthogonal product basis for $d_i \geq n-1$; where d_i is the dimension of the individual subsystems and n is the number of parties. In [24, 26], they considered the multipartite construction in $d \otimes d \otimes d$ but there the party ‘C’ can always define a nontrivial, nondisturbing measurement. Recently, in [29] they considered a small set of locally indistinguishable multipartite orthogonal product states but that sets can not be extended to a complete orthogonal product basis. In [30], they constructed a small set of only $2n$ members, which can be extended to a complete orthonormal product basis in arbitrary dimensions. In $(d)^{\otimes n}$ dimensions, there is also $2n$ such states which exhibits non-locality. Again, it is possible to construct a nonlocal orthogonal multipartite product basis, containing all $2n$ such states in $(d)^{\otimes n}$ dimensions (along with some other product states) and for such a class of bases, some information (not complete information of an unknown state of a given set), useful for state discrimination, can be extracted by performing nontrivial measurement by one or more parties without disturbing the orthogonality of the post measurement states. Now, this particular feature of extracting information is not welcome to manifest stronger form of nonlocality. The detailed discussion of which (taking an example) is given in a later portion of this paper.

So, it is still relevant to study about the structure of locally indistinguishable completable small set of multipartite orthogonal product states in $(d)^{\otimes n}$ dimensions, that also satisfies the above mentioned condition to manifest stronger form of nonlocality. There is another direction of research and that is to explore entanglement as a resource to distinguish nonlocal sets of quantum states [18–20].

The remaining portion of this paper is arranged as follows. In section (II), some mathematical definitions are given which are essential to explain the present work. In section (III), the construction of locally indistinguish-

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able small set of multipartite orthogonal product states in $(d)^{\otimes n}$ dimensions, is given. In section (IV), entanglement assisted discrimination of such a nonlocal set, is given and finally in section (V) the conclusion is drawn.

II. SOME USEFUL DEFINITIONS

Definition 1: A set of bipartite orthogonal product states are LOCC indistinguishable if no party is able to begin with a nontrivial, nondisturbing measurement [26]. Similarly, for multipartite system, if all of the parties are not able to begin with a nontrivial nondisturbing measurement then a set of multipartite orthogonal product states, is a LOCC indistinguishable set [29].

Definition 2: (Nontrivial measurement) In quantum mechanics, a measurement is defined by a set of positive operator-valued measure (POVM) elements. These elements satisfy the completeness relation, i.e., the sum of all these POVM elements must be equal to identity. Now, if all of the POVM elements are proportional to the identity operator, then it is called a trivial measurement because these type of measurements are unable to extract any information about the quantum state. On the other hand, if some of the POVM elements are not proportional to identity then it is called a nontrivial measurement.

Definition 3: (Nondisturbing measurement) If there is a set of pairwise orthogonal quantum states and a measurement, defined in such a way, that after performing the measurement, the post measurement states are also pairwise orthogonal to each other. In other words, these type of measurements are basically orthogonality preserving measurement.

Definition 4: An incomplete set of orthogonal product states, is called completable set in a Hilbert space, if it can be extended to a complete orthonormal product basis in the same Hilbert space.

For local distinguishability, it is necessary that atleast one party is able to begin with a nontrivial and nondisturbing measurement [13, 14, 17].

III. CONSTRUCTION

For better understanding, first an example in $3 \otimes 3 \otimes 3$ is given, then the small set in $(d)^{\otimes n}$ dimensions, is presented. Here, $|i \pm j\rangle = \frac{1}{\sqrt{2}}(|i\rangle \pm |j\rangle)$ for $1 \leq i, j \leq 3$.

Example.1: In $3 \otimes 3 \otimes 3$, following 12 states are locally indistinguishable.

$$\begin{aligned} |\phi_{1,2}\rangle &= |1\rangle|2\rangle|1 \pm 2\rangle & |\phi_{3,4}\rangle &= |1\rangle|3\rangle|1 \pm 3\rangle \\ |\phi_{5,6}\rangle &= |2\rangle|1 \pm 2\rangle|1\rangle & |\phi_{7,8}\rangle &= |3\rangle|1 \pm 3\rangle|1\rangle \\ |\phi_{9,10}\rangle &= |1 \pm 2\rangle|1\rangle|2\rangle & |\phi_{11,12}\rangle &= |1 \pm 3\rangle|1\rangle|3\rangle \end{aligned} \quad (1)$$

Proof: To distinguish these states via LOCC, at least one of the three parties has to begin with a nontrivial and orthogonality preserving measurement. In other words,

all the POVM elements should not be proportional to identity and the post measurement states must be pairwise orthogonal to each other, making further discrimination feasible.

Suppose, the first party goes to begin the measurement on his (her) subsystem. The measurement elements $M_m^\dagger M_m$, can be written in the basis $\{|1\rangle, |2\rangle, |3\rangle\}_1$:

$$M_m^\dagger M_m = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

After the measurement, all the states $\{M_m \otimes I \otimes I|\phi_i\rangle, i = 1, \dots, 12\}$ are pairwise orthogonal to each other. Now, consider $|\phi_{5,7}\rangle$, taking the inner product of the post measurement states for those two states, $\langle\phi_5|M_m^\dagger M_m|\phi_7\rangle = \langle\phi_7|M_m^\dagger M_m|\phi_5\rangle = 0$; which implies $\langle 2|M_m^\dagger M_m|3\rangle = a_{23} = \langle 3|M_m^\dagger M_m|2\rangle = a_{32} = 0$. Similarly, considering $|\phi_{1,5}\rangle$, it is possible to show $a_{12} = a_{21} = 0$ and for $|\phi_{3,7}\rangle$, $a_{13} = a_{31} = 0$. Thus all the off diagonal terms of the above matrix are zero. Now, consider the inner product of the post measurement states of $|\phi_{9,10}\rangle$, which implies that $\langle\phi_9|M_m^\dagger M_m|\phi_{10}\rangle = \langle 1 + 2|M_m^\dagger M_m|1 - 2\rangle = 0$ and thus, $a_{11} = a_{22}$. Similarly, for the states, $|\phi_{11,12}\rangle$, it is possible to show that $a_{22} = a_{33}$. Thus, $a_{11} = a_{22} = a_{33}$. Hence, all the measurement elements are proportional to identity. So, the 1st party is not able to begin with a nontrivial and orthogonality preserving measurement.

If, the 2nd party wish to begin with a nontrivial and orthogonality preserving measurement then the same condition is applied and that is, after the measurement, all the states $\{I \otimes M_m \otimes I|\phi_i\rangle, i = 1, \dots, 12\}$ are pairwise orthogonal to each other. Now to show that the off diagonal terms are zero, consider the pair of states $|\phi_{1,9}\rangle$, $|\phi_{1,3}\rangle$, $|\phi_{3,11}\rangle$ separately, and it is possible to show that $\langle i|M_m^\dagger M_m|j\rangle = 0$, with $i, j = 1, 2, 3$ and $i \neq j$. Considering the pair of states, $|\phi_{5,6}\rangle$ and $|\phi_{7,8}\rangle$, it is possible to show that all diagonal terms are equal and therefore, the measurement elements are proportional to identity. So, the 2nd party is not be able to begin with a nontrivial and orthogonality preserving measurement.

Now if, the 3rd party wish to begin with a nontrivial and orthogonality preserving measurement then the post measurement states $\{I \otimes I \otimes M_m|\phi_i\rangle, i = 1, \dots, 12\}$ are pairwise orthogonal to each other. Now to show that the off diagonal terms are zero, consider the pair of states $|\phi_{5,9}\rangle$, $|\phi_{7,11}\rangle$, $|\phi_{9,11}\rangle$ separately and it is possible to show that $\langle i|M_m^\dagger M_m|j\rangle = 0$, with $i, j = 1, 2, 3$ and $i \neq j$. Considering the pair of states, $|\phi_{1,2}\rangle$ and $|\phi_{3,4}\rangle$, it is possible to show that all diagonal terms are equal and therefore, the measurement elements are proportional to identity. So, the 3rd party is not be able to begin with a nontrivial and orthogonality preserving measurement.

Hence, it is proved that no party is able to start with a nontrivial and nondisturbing measurement and thus the 12 states of equation (1) are LOCC indistinguishable. Now, consider the following set of product states: (in $3 \otimes 3 \otimes 3$ dimensions)

$$\begin{aligned}
&|1\rangle|1\rangle|1\rangle \quad |1\rangle|2\rangle|3\rangle \quad |1\rangle|3\rangle|2\rangle \quad |2\rangle|1\rangle|3\rangle \quad |2\rangle|2\rangle|2\rangle \\
&|2\rangle|2\rangle|3\rangle \quad |2\rangle|3\rangle|1\rangle \quad |2\rangle|3\rangle|2\rangle \quad |2\rangle|3\rangle|3\rangle \quad |3\rangle|1\rangle|2\rangle \quad (2) \\
&|3\rangle|2\rangle|1\rangle \quad |3\rangle|2\rangle|2\rangle \quad |3\rangle|2\rangle|3\rangle \quad |3\rangle|3\rangle|2\rangle \quad |3\rangle|3\rangle|3\rangle
\end{aligned}$$

The states of equation (1) and that of equation (2) form a orthogonal product basis (say, basis-1) in $3 \otimes 3 \otimes 3$. So, the small set, given in equation (1) is extendible to a complete orthonormal product basis, i.e, the set is completable.

One can argue that the set of 6 states $|\phi_{1,2,5,6,9,10}\rangle$ of equation (1) are enough to show local indistinguishability, then what is the importance of the other 6 states $|\phi_{3,4,7,8,11,12}\rangle$. It is true that the states $|\phi_{1,2,5,6,9,10}\rangle$ are LOCC indistinguishable [30] but that does not mean that any product basis in $3 \otimes 3 \otimes 3$, containing all 6 six states (along with some other product states) satisfies the condition that no party is be able to begin with a nontrivial and nondisturbing measurement in 3×3 . In fact, it is possible to construct a nonlocal product basis in $3 \otimes 3 \otimes 3$, containing all 6 states $|\phi_{1,2,5,6,9,10}\rangle$, but that product basis may exhibit weaker form of nonlocality in the sense, that some information, useful for state discrimination can be extracted by performing a nontrivial and nondisturbing measurement, details of which is given below. Consider the following set of product states:

$$\begin{aligned}
&|1\rangle|1\rangle|1\rangle \quad |1\rangle|1\rangle|3\rangle \quad |1\rangle|2\rangle|3\rangle \quad |1\rangle|3\rangle|1\rangle \quad |1\rangle|3\rangle|2\rangle \\
&|1\rangle|3\rangle|3\rangle \quad |2\rangle|1\rangle|3\rangle \quad |2\rangle|2\rangle|2\rangle \quad |2\rangle|2\rangle|3\rangle \quad |2\rangle|3\rangle|1\rangle \\
&|2\rangle|3\rangle|2\rangle \quad |2\rangle|3\rangle|3\rangle \quad |3\rangle|1\rangle|1\rangle \quad |3\rangle|1\rangle|2\rangle \quad |3\rangle|1\rangle|3\rangle \quad (3) \\
&|3\rangle|2\rangle|1\rangle \quad |3\rangle|2\rangle|2\rangle \quad |3\rangle|2\rangle|3\rangle \quad |3\rangle|3\rangle|1\rangle \quad |3\rangle|3\rangle|2\rangle \\
&\quad \quad \quad |3\rangle|3\rangle|3\rangle
\end{aligned}$$

The product states of the above equation, and the product states $|\phi_i\rangle$, ($i = 1,2,5,6,9,10$) together form a product basis (say, basis-2) in $3 \otimes 3 \otimes 3$. But here it is possible to define a nontrivial and orthogonality preserving measurement via which all three parties are able to distinguish between two sets, where one set is containing only 8 states $|\phi_i\rangle$, ($i = 1,2,5,6,9,10$), $|1\rangle|1\rangle|1\rangle$, $|2\rangle|2\rangle|2\rangle$ and the other is containing the states, given in equation (3) except two states $|1\rangle|1\rangle|1\rangle$, $|2\rangle|2\rangle|2\rangle$. Consider a two-outcome measurement with the following projectors:

$$P_1 = |1\rangle\langle 1| + |2\rangle\langle 2|, \quad P_2 = |3\rangle\langle 3| \quad (4)$$

This measurement can be performed by all three parties. In fact, if the measurement outcome of any of the parties, is ‘2’ then the perfect discrimination is possible. On the other hand, the basis-1 containing all the product states of equation (1), so, no party will be able to begin with a nontrivial and orthogonality preserving measurement. So, basis-1 shows stronger form of nonlocality than basis-2. Here is the importance of the small set of equation (1) and it is also fruitful to generalize such a set for

$(d)^{\otimes n}$ dimensions to form similar class of nonlocal bases as basis-1 in $(d)^{\otimes n}$ with different values of ‘d’.

Theorem 1: The set of $2n(d-1)$ pairwise orthogonal n -party product states in $(d)^{\otimes n}$ dimensions, given in equation (5) are not perfectly distinguished by LOCC.

[For these set of states, $i = 2\dots d$, $k = (i-1)$, each block contains $2(d-1)$ states, and there are total n blocks, consistent with the n number of parties. Here, $|a \pm b\rangle = \frac{1}{\sqrt{2}}(|a\rangle \pm |b\rangle)$ for $1 \leq a, b \leq d$. These set of product states can be extended to a complete orthonormal product basis in $(d)^{\otimes n}$ dimensions, as the set of states does not include any ‘stopper’ state [11, 12].]

Proof: To prove that the product states of equation (5) are not perfectly locally distinguishable, one can follow the same proof technique as mentioned before. That is, if no party is able to begin with a nontrivial, orthogonality preserving measurement, then the set of states are not perfectly locally distinguishable.

In brief, to prove the local indistinguishability of the above mentioned set, it is required to show that whenever a party wish to measure his (her) subsystem, the measurement elements are proportional to a $d \times d$ identity matrix. (All the diagonal terms of the matrix form of the measurement elements are equal to each other and the off diagonal terms of the same are all zero.)

$ \phi_k\rangle = 11\dots 1\rangle i\rangle 1+i\rangle$	(5)
$ \phi_{(d-1)+k}\rangle = 11\dots 1\rangle i\rangle 1-i\rangle$	
$ \phi_{2(d-1)+k}\rangle = 1\dots 1\rangle i\rangle 1+i\rangle 1\rangle$	
$ \phi_{3(d-1)+k}\rangle = 1\dots 1\rangle i\rangle 1-i\rangle 1\rangle$	
$ \phi_{4(d-1)+k}\rangle = 1\dots 1\rangle i\rangle 1+i\rangle 11\rangle$	
$ \phi_{5(d-1)+k}\rangle = 1\dots 1\rangle i\rangle i-i\rangle 11\rangle$	
\vdots	
$ \phi_{(2n-4)(d-1)+k}\rangle = i\rangle 1+i\rangle 11\dots 1\rangle$	
$ \phi_{(2n-3)(d-1)+k}\rangle = i\rangle i-i\rangle 11\dots 1\rangle$	
$ \phi_{(2n-2)(d-1)+k}\rangle = 1+i\rangle 11\dots 1\rangle i\rangle$	
$ \phi_{(2n-1)(d-1)+k}\rangle = i-i\rangle 11\dots 1\rangle i\rangle$	

Now, suppose that the $(n-1)th$ party wish to begin with a nontrivial and non-disturbing measurement. Consider the states $|\phi_k\rangle$ states, the post measurement states, $\{I \otimes I \otimes \dots \otimes M_m \otimes I|\phi_k\rangle\}$ are pairwise orthogonal to each other. Because of the orthogonality of the states, $\langle \phi_j | I \otimes I \otimes \dots \otimes M_m^\dagger M_m \otimes I | \phi_k \rangle = 0$, or, $\langle j | M_m^\dagger M_m | k \rangle = 0$, which implies $a_{jk} = 0$ with $j, k = 2\dots d$ and $j \neq k$. These a_{jk} ’s are basically the off diagonal terms of the matrix form of the measurement elements of the $(n-1)th$ party. Next, consider the states $|\phi_k\rangle$ and $|\phi_{(2n-2)(d-1)+k}\rangle$ and the inner product of $\langle \phi_{(2n-2)(d-1)+k} | I \otimes I \otimes \dots \otimes M_m^\dagger M_m \otimes I | \phi_k \rangle$ which is equal to zero, implies that $\langle 1 | M_m^\dagger M_m | i \rangle = a_{1i} = 0$. Therefore,

all the off diagonal terms are zero. Now, to prove that the diagonal terms are equal to each other consider the states, $|\phi_{2(d-1)+k}\rangle$ and $|\phi_{3(d-1)+k}\rangle$ and consider the inner product $\langle\phi_{2(d-1)+k}|I \otimes I \otimes \dots \otimes M_m^\dagger M_m \otimes I|\phi_{3(d-1)+k}\rangle = 0$, implies that $\langle 1+i|M_m^\dagger M_m|1-i\rangle = 0$, or $a_{11} = a_{ii}$. Therefore, it is established that the matrix form of the measurement elements for $(n-1)th$ party is proportional to a $d \times d$ identity matrix. So, $(n-1)th$ party is not able to start with a nontrivial and nondisturbing measurement. Following the same technique, it is possible to show that no party is able to begin with a nontrivial and orthogonality preserving measurement in $d \times d$ dimensions. Therefore, the product states, given in the above equation are locally indistinguishable. [If there are only two parties then, it is possible to construct a set of $4d-4$ locally indistinguishable states in $d \times d$, explicit construction of which is given in the paper [26]. According to Walgate *et al.*, in 2×2 , construction of locally indistinguishable product basis, is not possible [14]. But for three and more parties, each holding a qubit, it requires only $2n$ -states to exhibit similar form of nonlocality in $(2)^{\otimes n}$ dimensions, imposing the same condition on the measurement in 2×2 , in fact, the states are same as given in [30].]

[In this work, only a class of completable nonlocal set of orthogonal product states, is considered in $(d)^{\otimes n}$ dimensions. But following the same construction it is also possible to construct a nonlocal set of fewer orthogonal product states in the same $(d)^{\otimes n}$ dimensions. For example, consider the states $|\phi_{l(d-1)+k}\rangle$ with $l = 1, 3, 5, \dots, (2n-1)$ from the equation (5) and then together with all these states, add a stopper state $|1+2+\dots+d\rangle|1+2+\dots+d\rangle\dots|1+2+\dots+d\rangle$. It is easy to show that for this kind of set of multipartite orthogonal product states, no party is able to begin with a nontrivial and nondisturbing measurement in $d \times d$ dimensions. So, these sets are locally indistinguishable. It is also important to remember that as this class of sets of multipartite orthogonal product states in $(d)^{\otimes n}$ includes a 'stopper' state, so, the stopper state will force the set to be uncompletable in the same $(d)^{\otimes n}$ dimensions. Thus, it is also an example of a class of uncompletable sets. These sets are including only $n(d-1)+1$ multipartite orthogonal product states in $(d)^{\otimes n}$ dimensions.]

IV. DISCRIMINATION

In this section, entanglement assisted discrimination of the set of equation (5), is considered. The protocol which is given here, is similar as that of [19]. Suppose, there is a 2×2 maximally entangled state $|00\rangle + |11\rangle$ (the normalization factors are ignored here), shared between the $(n-1)th$ and nth party. The states of the equation (5), together with the resource, have the form, given in equation (6). *Protocol:* Firstly, the nth party performs a two-outcome measurement. The projectors, corresponding to the measurement, are given by $R_1^{(n)} =$

$$|10\rangle\langle 10| + |i1\rangle\langle i1|, R_2^{(n)} = |11\rangle\langle 11| + |i0\rangle\langle i0|.$$

$ 11\dots 1\rangle[i0\rangle 10 \pm i0\rangle + i1\rangle 11 \pm i1\rangle]$	(6)
$ 1\dots 1\rangle i\rangle[10 \pm i0\rangle 10\rangle + 11 \pm i1\rangle 11\rangle]$	
$ 1\dots 1\rangle i\rangle 1 \pm i\rangle[10\rangle 10\rangle + 11\rangle 11\rangle]$	
\vdots	
$ 1\rangle i\rangle 1 \pm i\rangle 1\dots 1\rangle[10\rangle 10\rangle + 11\rangle 11\rangle]$	
$ i\rangle i \pm i\rangle 1\dots 1\rangle[10\rangle 10\rangle + 11\rangle 11\rangle]$	
$ 1 \pm i\rangle 1\dots 1\rangle[10\rangle i0\rangle + 11\rangle i1\rangle]$	

If, the outcome '1' occurs, then the states of the equation (6), are transformed to another set of states, given in equation (7).

$ 11\dots 1\rangle[i0\rangle 10\rangle + i1\rangle i1\rangle]$	(7)
$ 1\dots 1\rangle i\rangle 10 \pm i0\rangle 10\rangle$	
$ 1\dots 1\rangle i\rangle 1 \pm i\rangle 10\rangle 10\rangle$	
\vdots	
$ 1\rangle i\rangle 1 \pm i\rangle 1\dots 1\rangle 10\rangle 10\rangle$	
$ i\rangle i \pm i\rangle 1\dots 1\rangle 10\rangle 10\rangle$	
$ 1 \pm i\rangle 1\dots 1\rangle 11\rangle i1\rangle$	

Now, $(n-1)th$ party does a two-outcome measurement, the projectors are given by- $R_1^{(n-1)} = |11\rangle\langle 11|$, $R_2^{(n-1)} = |10\rangle\langle 10| + |i0\rangle\langle i0| + |i1\rangle\langle i1|$. Here, if the outcome '1' occurs then the last block from the equation (7) gets eliminated. The states of the last block can be further discriminated by two simple steps: nth party does a simple $(d-1)$ -outcome projective measurement and corresponding to each outcome, there are two orthogonal states left, which can be further discriminated by some local protocol according to Walgate *et al* [1]. Again, we go back to the measurement by the $(n-1)th$ party, if the outcome '2' occurs, then there are $(n-1)$ blocks. Each of the blocks get eliminated on performing a two-outcome measurement by each party. For example, if the 1st party performs, defining two projectors, $Q_1^{(1)} = |1\rangle\langle 1|$, $Q_2^{(1)} = \sum_i |i\rangle\langle i|$, on occurrence of '1' there are $(n-2)$ blocks left and on occurrence of '2' the 1st party measures again with a simple $(d-1)$ outcome projective measurement and corresponding to each outcome, there are two orthogonal states left, which can be further discriminated by some local protocol as mentioned before. This process is repeated by 1 to $(n-2)th$ party. Now, there is left only the 1st block. For the states of the 1st block, $(n-1)th$ party performs a $(d-1)$ outcome projective measurement, defined by- $Q_i^{(n-1)} = |i0\rangle\langle i0| + |i1\rangle\langle i1|$, and corresponding to each 'i', there are two orthogonal

states and they can be discriminated further. Now, we go back to the very early stage of this protocol, if the outcome ‘2’ occurs due to the measurement by n th party, then there are another set of states as given in equation (7) and these states can be discriminated in the similar fashion as mentioned above. Here, the protocol ends.

V. CONCLUSION

In this paper, a class of nonlocal set of multipartite orthogonal product states, is shown. This is important for the better understanding of the phenomenon ‘nonlocality without entanglement’ in multipartite case. A protocol is also given for the entanglement assisted discrimination

of such a class of states. Future directions may include if it is possible to construct smaller set (than the present one) of multipartite product states that are locally indistinguishable, imposing the same condition on the measurement in $d \times d$ and of course to find entanglement assisted local protocol that is sufficient to distinguish such a set of product states.

ACKNOWLEDGEMENT

The author acknowledges the financial support from the council of scientific and industrial research (CSIR), Government of India.

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